Computational Choice: The Reasons Behind the Choices

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This paper represents some of the results of a study carried out to explore the computational choices made by 75 students in Years 5-7. Data were collected to determine the initial and subsequent computational choices made by students. A record was made of the success rate for various computational choices and students were interviewed to determine the reasons for making particular computational choices. Reasons given by students were placed in four categories of number magnitude, efficiency, knowledge of multiplication facts, and teacher influence.

Introduction

The ability to choose and use a repertoire of computation methods is an important goal of the mathematics curriculum (Australian Education Council, 1991). While this may seem to be a reasonable expectation for primary school mathematics programs, there is not a great deal of evidence to describe how students go about making decisions relating to computational choice. Various models of computational choice have been suggested (National Council of Teachers of Mathematics, 1989; Trafton, 1994; Swan & Bana, 1998), but few have been tested. Seventy-five students in Years 5-7 in two schools in Western Australia were interviewed, based on their responses to an eighteen-item instrument in order to evaluate the computational choice model postulated by Swan and Bana (1998). The results were collated and an analysis was undertaken to determine the patterns of computational choices made by students, the success rates of these choices, and also the reasons given for such choices. The main focus of this paper is on the reasons given by students for making particular computational choices.

Background

Previous studies (Reys, Reys & Hope, 1993; Price 1997) have tended to focus on the computational choices made by students, but not on their reasons for making these choices. Reys, Reys and Hope (1993) found that while written methods did tend to dominate the thinking of many students, mental methods were also employed in many cases, with calculator methods being the least used. While they did not interview the students to determine reasons for particular computational choices, they did consider the nature of the questions and noted that computational choice seemed to be related to the nature of the numbers used.

Price (1997) suggested that teacher presence was a strong variable in computational choice. This was explained by the idea that teachers generally seem to promote written methods of computation ahead of mental and calculator methods. He designed a teaching experiment which sought to test this notion and found that with a teacher present, students chose written methods 56 percent of the time, calculators 26 percent of the time, and mental methods 19 percent of the time. A swing of 10 percent from written to calculator methods was noted when the teacher left the room.

Several authors (Reys & Reys, 1998; Kamii, 1994) have argued that an over-reliance on written methods of calculation interferes with the development of number sense. Thus the argument in favour of increased mental and calculator methods is that they develop a better understanding of numbers, generally referred to as number sense. The rather narrow view of computation as the development of standard written algorithms has certainly been challenged (Ralston, 1999; Reys & Nohda, 1994; Trafton 1994). A model of computational choice (NCTM, 1989, p. 9) outlining three main approaches (mental, written and calculator methods) to computation was refined by Trafton (1994) to emphasise the role of estimation in computational choice. Ruthven (1998) challenged this simple linear model of computational choice stating that "... a refinement of the common-sense trichotomy between mental, written and calculator methods was necessary to take better account of different forms and functions of writing within computation" (pp. 29-30). Swan and Bana (1998) developed an alternative model of computational choice in an attempt to better describe its complex nature.

Methodology

Seventy-five students from two schools (one State and one Catholic) in the south-west of Western Australia were asked to solve eighteen computation items, using any methods they chose. An individual, clinical-interview approach was used with each student in the study. For each item the student was observed undertaking the computation; the researcher noted the chosen method(s) of computation; then the student was asked to explain how he/she solved the problem. The interview was audio taped, and the student's written jottings or calculations were collected to confirm observations made by the researcher. For each item the student was also asked why he/she chose a particular computational method. Once the student had completed the item using the preferred method(s) of solution and had explained the reasons for using that particular method, he/she was asked if they could use an alternative method of solution. The interviewer continued to ask the student for alternative methods of solution until he/she ran out of methods.

Results

The instrument and some of the preliminary results of the study have been reported elsewhere (Swan & Bana, 1999). Further preliminary results below present an overview of the choices made by students in relation to particular sets of items, then focus on the reasons given by students for the computational choices that they made.

Table 1 shows that overall, 35 percent of all calculations were performed using mental methods, 27 percent were undertaken using pencil and paper, and 27 percent were performed with a calculator. The remaining 11 percent of calculations involved either mixed methods or no method of solution at all. For most students there was little or no hesitation when making the choice as to which method to use when solving an item. There was little evidence to support the notion that students carefully examine a question before choosing a computational method. On a few occasions it was observed that students, having embarked on a particular method, found that it was inappropriate and abandoned it in favour of another.

Table 1
Percentages of Student Choices of Computational Methods (N= 75)

Mental	Written	Calculator	Mixed or Not Attempted
35	27	27	11

Table 2 lists the items where students preferred to use mental methods. These items involve small numbers, fractions or decimals, and also the two items that were presented in a shopping context. It may seem somewhat surprising to see mental methods as the preferred choice for most of the items involving fractions, but it appears that many students were unable to solve these types of questions using a paper-and-pencil algorithm and did not know how to use a calculator to assist them. Thus they only had one option left at their disposal. It also seemed that some students overestimated their ability to solve items using a particular computational method. For example, some seemed confident that they could solve an item using mental methods when in reality they experienced difficulty obtaining a solution.

Table 2

Items where Mental Methods were Preferred

Item Number	Item Content
1	28 + 37
11	$^1/_2 + ^3/_4$
12	$10 - 4\ ^3/_4$
13	$^2/_3$ of 45
14	$1.99 + $ 1.99 (presented in a shopping context)
15	$4.93 + 39c (presented in a shopping context)
16	7.41 – 2.5
18	3.5 ÷ 0.5

Table 3 lists the items that students preferred to solve with paper and pencil. The four items in this category are two-digit multiplication and subtraction computations which closely resemble the types of questions typically given in mathematics textbooks. Students generally spend a great deal of classroom time completing questions of this type. It appears that the cognitive demands of such two-digit items are such that many students in Years 5-7 are unable to complete the computation mentally and therefore need to use another method. Standard written algorithms are designed to be completed by recording interim steps, thus making them inefficient as a basis for mental methods.

Table 3

Items where Written Methods were Preferred

Item Number	Item Content
2	74 – 36
4	36 x 25
6	29 x 31
7	33 x 88

Table 4 shows the five items in which calculator use was the preferred method. "Big numbers", in some students' words, was a trigger that had students reach for a calculator. This was particularly evident in both Item 8 and in Item 17. The use of calculators dominated computational choice in Item 17, possibly because it involved both large numbers and decimals, and it was an unfamiliar calculation to many of the participants. Item 9, which involved the use of percentages was another item unfamiliar to many of the students, with a majority stating that they were unsure of how to proceed. Some of the students who were unsure of what to do, chose to use a calculator simply because they knew it had a percentage button. Many did not know how to use the percentage button and simply assumed that pressing the button in conjunction with the 750 would produce the desired result. Thirty-six percent of students chose to use a calculator method to solve 14 x 9 ÷ 6. This item often saw the use of combined methods of solution, with the first part being solved using one method and the second division part by another. It seems that these students chose to use a calculator because they were unfamiliar with this type of multi-step computation.

Table 4

Items where Calculator Methods were Preferred

Item Number	Item Content
3	369 ÷ 3
8	1000 x 945
9	10% of 750
10	14 x 9 ÷ 6
17	0.25 x 800

It appears that some aspects of a computation can override other considerations when it comes to making an initial choice of what method to use. For example, the "big number" cue seems to have come to the fore in Item 8 for 1000 x 945, where most students chose to use a calculator to complete what could be considered to be a relatively straight-forward mental calculation.

Reasons Given for Making Particular Choices

Previous research on computational choice by Reys, Reys and Hope (1993) reported on the computational preferences of students in Years 5-7, but the students were only required to state their preferences. The researchers indicated the need to interview students to find out why particular choices were made. Students participating in the current research were asked about the reasons for making particular computational choices and were also observed using their preferred method to undertake the computation.

The students in this study were not always able to clearly explain the reasons behind the computational choices they made. Some had problems articulating their reasons, while some appeared to automatically adopt a particular approach without much conscious thought. While some students were able to give detailed reasons for making a particular choice, others made broad statements such as "big numbers" to explain why they used a calculator. Even though a number of students gave their reason as "big numbers", it was most evident that students providing this explanation possessed differing viewpoints as to what constituted a big number. Similarly, other reasons given by students in the study tended to have various shades of meaning. The various reasons given for choosing a particular computational strategy are grouped under four common themes below, with examples from transcriptions of taped interviews with the students. This addition of comments made by the students helps to clarify what was meant when a particular reason was given.

Magnitude of Numbers

Students tended to use mental computation when the numbers involved were small, but often cited "big numbers" as the reason for choosing not to use mental methods. In some cases this observation prompted students to use a written method, while in other cases they used a calculator. In the following extract the student elaborates on the "big number" explanation. Note how the student links the idea of "big numbers" to memory constraints by suggesting they are difficult to remember. It is possible that the cognitive load is increased by the introduction of larger numbers.

I: Water noodles costs $4.93 and two-minute noodles sell for 39c, how much would that cost me altogether?

S: I'd just go $4.93, plus 39c equals, 9 plus 3 threes 2 left so carry the 1, 9 plus 3 is the same, carry the 1, put 2 and that's $5.22.

I: And why did you choose to do that by writing it down?

S: Because $4.93 is a big number to remember. You might just forget and put $4.39, it's better just to write it down so you can remember it properly.

Here is another explanation.

I: What about 369 ÷ 3?

S: 123

I: So you used the calculator there, why?

S: Because it's a big number, it's easier to use a calculator.

I: Could you do that in your head?

S: Yes, I'd do it like I was writing it in my head.

There was no evidence to suggest this student could successfully complete the division item in her head but the comment about "writing it in her head" is interesting. Several students described mental methods based on the use of a typical paper-and-pencil approach. The usefulness of this approach will be discussed later.

Some cautionary remarks need to be made about the "big number" explanation for using a calculator. Students using this reason for adopting a calculator-based approach to solving the item were not totally reliant on the use of a calculator but simply chose it as the most expedient option. The following extract shows an example of a student who chose to use calculator but was also able to complete the same item mentally. Whether the initial use of the calculator made the mental computation simpler is debatable, as her explanation of how to solve the item mentally is most plausible.

I: 70 x 600

S: 42 000

I: And you used a calculator for that one – why?

S: Because it was a big number.

I: Could you have done it another way?

S: I could have gone seven times six is 42 and then put three zeros on it.

I: Would you have done it in a written way at all?

S: No.

Later the same student also used a calculator to solve 1000 x 945. Note the reason she gave for using a calculator and her later explanation of how she would solve the same item mentally.

I: 1000 x 945

S: 945 000

I: Why did you do that one on a calculator?

S: Because I don't really know how to do long multiplication.

I: Could you do that one mentally?

S: Yes, I could have gone 945 x 1 and add three zeros.

The technique of crossing off and adding zeros was used by a number of students for Item 5 and Item 8. In most cases students experienced difficulty applying this strategy and did not appear to have any real understanding of why it worked.

Efficiency

A common reason given for using a calculator involved speed. Expressions like "it's faster", "it's quicker", or "it's quick and easy" were often given in support of calculator use. Speed was also cited as a reason for not using the written algorithm. Comments such as "it would take longer" and "it would take too long" were commonly used to explain why the written algorithm was not favoured. Examples of students using this reason for various items are given below. In the first example the student uses the "big number" reason in conjunction with a comment about the amount of time it would take to complete the item on paper to explain why he opted to use a calculator. The inflection in his voice suggested that the speed factor weighed heavily on his mind.

I: 1000 x 945

S: 945 000

I: Okay, I notice that you used a calculator, why was that?

S: Big sum.

I: Could you do it any other way?

S: On paper, but it would take forever.

I: Any other way?

S: No.

Knowledge of Multiplication Facts

The comments made by students in the study indicate that basic multiplication facts or "tables" feature prominently in their thinking about mathematics and have an influence on the computational choices that they make. The following comments by a student indicate a preoccupation with "tables". The last comment is of concern as it appears as though the student considers that, to perform such a multiplication requires knowledge of basic facts beyond 9 x 9.

I: 14 x 9 ÷ 6

S: 21

I: You used a calculator, why?

S: Because 14 x 9 and divided by 6, I don't know my times table up to that standard.

Here is another case.

I: What about 36 x 25?

S: 900

I: Once again you chose to use a calculator. Why?

S: Because again it was a big number. I don't really know my 36 times table.

Teacher Influence

The comments made by students in the interviews help to answer the question "What influences student choice of computation method?" There were comments from students alluding to the fact that they did it this way in class. For example the influence of the teacher and the teaching of standard written algorithms may be noted in the following extract.

I: All right but you prefer to write it down. Is there a reason for that?

S: I just was taught to do it and I've always been doing it that way. It's easier because when I do it on the calculator I think it's not using your brain and you don't work things out better and you don't get smarter, so I write it down because it's easier and I understand better.

The student also indicates an aversion to using the calculator, based on the notion of "not getting any smarter" as a result of using a calculator. The impact of this thinking is to restrict computational choice.

Conclusion

The examples above represent only a small sample of the responses that were given by students in the study, but they do serve to throw some light on the reasons for students employing particular computational approaches. As Price (1997) has found previously, teachers have a strong influence over the computational choice made by children. Schools and teachers chosen to be involved in this research were in part selected because they allowed students to keep calculators in their desks and because they espoused contemporary beliefs about the need to develop a repertoire of computational strategies. Despite this, students still referred to the teacher having an impact on their computational choice. Clearly there are implications for teachers and teacher educators in this result.

The issue of speed may also be influenced by classroom practice where students can be encouraged to complete as many questions as they can in the time available. It was not surprising to find students mentioning "tables" as a reason for making a particular computational choice, because significant adults in students' lives such as parents and grandparents may often refer to "tables" whenever any mention is made of mathematics. Basic number facts drill is a common practice in many classrooms and, while it was not a significant part of the daily routine in the schools included in this study, there was evidence of "tables" practice having been carried out in the classrooms. The "big numbers" explanation was a broad category used by students to explain the choice of written or calculator methods. Although this explanation was not used consistently by all students in the study, this is possibly due to their wide range of experiences.

Further research is needed to examine the various influences on students' computational choice, particularly that of their previous school mathematics programs. Teachers need to be assisted to help their students to make appropriate computational choices. Shumway (1994) refers to this idea as "metacomputation" where students would be encouraged to consider various aspects of a computational strategy before defaulting to a favoured approach. From the current research it appears, however, that students make a fairly hasty decision based on a limited set of criteria.

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